Transition to Turbulence in Two-Dimensional Wakes

Anthony Demetriades*
Ford Aerospace & Communications Corp., Newport Beach, Calif.

A new technique is advanced for predicting the location of the laminar turbulent transition zone in two-dimensional wake flows. The technique is based on postulating a minimum turbulence Reynolds number required for sustaining the turbulence once it occurs. Using existing wake turbulence data and compressible laminar wake theory, the turbulence Reynolds number is then algebraically expressed in terms of freestream or wake edge conditions and integral properties. An example is calculated using laminar wake similarity, and a simple formula is found by which the transition distance is linked to the stream Mach number, the heat transfer rate, ratio of specific heats, and other parameters. Some of the existing transition data are utilized to find that the minimum or threshold turbulence Reynolds number is about 15, whereas other available data are compared and found in good agreement with the resulting formula.

Nomenclature

B	= a constant, see Eq. (20)
C_1	= a constant in the fluctuation formula, Eq. (5)
C_2	= a constant in the scale formula, Eq. (6)
C_3	= C_1C_2
C_Dh	= momentum defect of wake
d	= diameter of wake-shedding body
k	= exponent in the viscosity-temperature relation, Eq. (11)
M	= Mach number
Q	= dimensionless thermal power exchanged, Eq. (16)
$\tilde{Q}$	= dimensional thermal power exchanged between body and flow
Re	= Reynolds number
Re_o	= minimum Reynolds number for transition
Re_w	= wake Reynolds number (based on edge properties and C_Dh)
Re_{do}	= defined by Eq. (9)
Re_Λ	= turbulence Reynolds number
$Re_{\Lambda o}$	= minimum possible turbulence Reynolds number
Re_{xT}	= Reynolds number based on edge properties and x_T
t	= temperature defect, $=[T(0)-T_e]/T_e$
T	= static temperature
u	= velocity
x	= distance from the wake-shedding body
$\bar{x}$	= nondimensional distance $=x/C_Dh$
γ	= ratio of specific heats
Λ	= integral scale of turbulence
ν	= kinematic viscosity
ρ	= density
σ	= Prandtl number
$(\ )'$	= rms wideband fluctuation
$(\ )_\infty$	= freestream properties
$(\ )_e$	= wake edge properties
$(\ )(0)$	= axis properties
$(\ )_T$	= properties at the transition point (zone)

I. Introduction

THE issue of laminar-turbulent transition in high-speed wake flows has drawn a great deal of attention since the late 1950s. During the 1960s, especially, there was considerable work done in supersonic and hypersonic wind tunnels and hypervelocity ballistic ranges to detect transition in wakes and to correlate its occurrence using the stream and model or projectile parameters. Since the transition question was and still remains theoretically intractable for all shear flows, these early efforts resulted in usable but empirical predictions offering little, if any, insight into the physics of the process.

A review of the wake transition literature† nevertheless reveals certain systematic dependencies of the transition process and its location on certain parameters. Briefly stated, these trends are as follows:

1) Whereas at low speeds (e.g., $M_\infty=0$) wakes are usually completely laminar or completely turbulent, at the higher Mach numbers the transition region can appear at some location downstream of, and often far from, the wake-shedding body.

2) Increasing the body Reynolds number moves transition forward; increasing M_∞ moves transition rapidly aft.

3) A minimum "critical" Reynolds number based on body size is required for transition to appear anywhere in the wake, and this minimum number increases rapidly as M_∞ increases.

4) At a fixed Mach number, the Reynolds number

$$Re_{xT}\equiv u_e x_T/\nu_e \tag{1}$$

appears to be constant. The constant was found to be of order 50,000-80,000 at $M_\infty=6$, where u_e and ν_e are taken to be adjacent to the transition region.

5) Heating the wake (or its shedding body) moves transition aft; cooling it moves transition forward.

Based on these observations, Lees[1] synthesized a picture of wake transition, shown here in Fig. 1, to explain the progress of x_T when the Reynolds number varies. The curve shown is a composite of calculation, speculation, and measurement and involves three distinct regions. Lees recognized that at very low Re (large Re^{-1}) viscosity dissipates turbulence so fast that, regardless of wake instability, no turbulence can sustain itself in the wake. Based on experiments by Behrens[5] and Kendall,[6] he surmised that this minimum critical Reynolds number

$$Re_m\equiv(u_\infty d/\nu_\infty)_m \tag{2}$$

would be of order 20,000 at most for very high M_∞. The second region, shown by a straight line in Fig. 1, represents

Received Sept. 12, 1977.

Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Boundary Layers and Convective Heat Transfer—Turbulent.

*Supervisor, Fluid Mechanics Section, Aeronutronic Division. Associate Fellow AIAA.

†A comprehensive list of wake transition data and empirical predictions of transition onset is contained in Ref. 1. Research activity in this problem has since declined and the few more recent works[2-4] have provided more data and data correlations rather than novel or detailed physical interpretations.

the conclusions drawn from the earlier experiments[7] by which R_{xT} was constant, at least at fixed M_∞ (observation 4, above). Finally, Lees invoked hydrodynamic stability theory to interpret the observation that near the wake "neck" the transition region advances upstream very slowly even after Re has considerably increased. Concurrent stability calculations by Gold[8] also provided support for the heating-cooling effect of transition as noted above. Nevertheless, stability theory can, at best, indicate only the direction in which the transition region is expected to move as opposed to its location.

The object of the present paper is to give a quantitative description of the location of a transition in a wake. This is done by using a simple rule that basically extends Lees' viscous dissipation estimates over the entire wake, with the possible exception of the near wake region. The roles of Reynolds number, Mach number, heat-transfer rate, and specific heat ratio are indicated algebraically and numerically. We shall limit ourselves to the two-dimensional wake geometry only, although both the concept and the calculations can easily be extended to other geometries as well.

II. Role of Turbulence Reynolds Number

By definition, transition will generate a region of turbulence downstream of it. Thus, if that turbulence cannot preserve itself, transition cannot occur in the accepted usage of the term. Now there are many, often redundant, criteria for the "preservation" of turbulence in the face of viscous dissipation; here we shall choose an elementary one, that the turbulence Reynolds number

$$Re_\Lambda \equiv u'\Lambda/\nu \tag{3}$$

must lie at or above a threshold value

$$(Re_\Lambda)_{\min} = Re_{\Lambda 0} \tag{4}$$

This statement embodies the sum total of the physics of our assertion. Its application is equally simple. For any given wake, a postulate is made as to where transition lies; Re_Λ is then computed in the turbulence just downstream of this postulated position. If the Re so found is smaller than $Re_{\Lambda 0}$, our postulate is wrong and another location is chosen until a location is found where Eq. (3) is just satisfied. Thus, the location where transition can occur is found by rejecting those locations where transition cannot possibly occur if criterion (4) is considered.

Our job will become not only possible but also much easier if we can express the quantities u', Λ, and ν analytically. This can be made possible if we relate these quantities to integrals of the fluid motion and if we utilize long-established experimental evidence on wake turbulence. The procedure will be demonstrated with our first example, which follows.

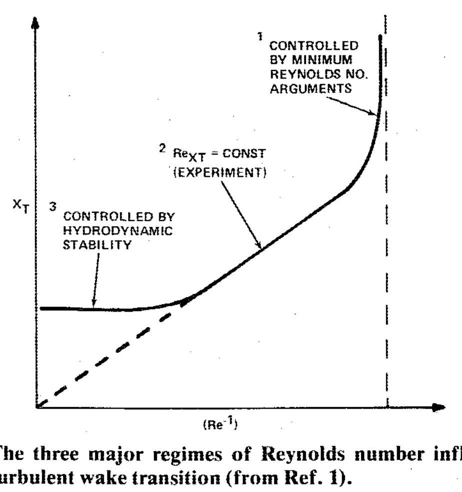

Fig. 1 The three major regimes of Reynolds number influence on laminar-turbulent wake transition (from Ref. 1).

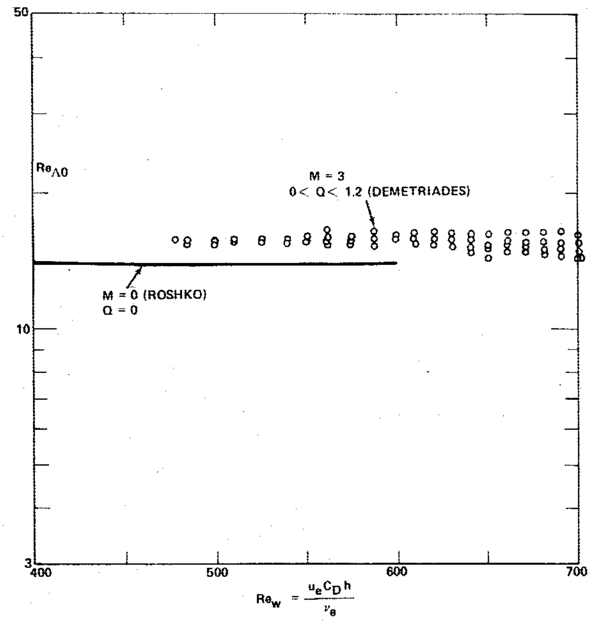

Fig. 2 Search for a threshold or minimum turbulence Reynolds number.

III. Two-Dimensional Incompressible Wake

Let us say that transition occurs somewhere downstream of the body in an incompressible two-dimensional wake (ν=const). The rms wideband velocity fluctuations u' on the axis of the turbulence region following transition‡ are known[9] to scale with the local velocity deficit $u_e - u(0)$:

$$u' = C_1[u_e - u(0)] = C_1[u_\infty - u(0)] \tag{5}$$

It is equally well established[9] that the integral turbulence scale Λ is related to the momentum defect $C_D h$ of the turbulent wake and the velocity defect $[u_\infty - u(0)]/u_\infty$ by

$$\Lambda = C_2 C_D h \frac{u_\infty}{u_\infty - u(0)} \qquad C_2 \text{ a constant} \tag{6}$$

The combination of Eqs. (3), (5), and (6) then gives a formula for the turbulence Reynolds number on the wake axis after transition has occurred:

$$Re_\Lambda = C_1 C_2 \left(\frac{u_\infty}{\nu_\infty} C_D h\right) = C_3 Re_w \tag{7}$$

where $C_3 = C_1 C_2$ is a numerical constant, and Re_w is recognized as the "wake Reynolds number" from which a "critical" or threshold wake Reynolds number can be defined:

$$Re_o \equiv Re_{\Lambda 0}/C_3 \tag{8}$$

The wake Reynolds number is constant for the entire wake because $C_D h$ is unchanged across the transition zone. For example, if the body is a cylinder of diameter d, then $C_D h = d$. Then if $Re_{\Lambda 0}$ is "universal," the cylinder wake would become turbulent when the cylinder Reynolds number attains a similarly universal value:

$$Re_{do} \equiv \left(\frac{u_\infty d}{\nu_\infty}\right)_o = \left(\frac{u_\infty C_D h}{\nu_\infty}\right)_o \equiv Re_o = Re_{\Lambda 0}/C_3 \tag{9}$$

Since the longitudinal distance x does not appear in it, this formula confirms observations (1) and (3) in the Introduction.

‡This scaling is valid for the dynamically equilibrated wake only.

The formula allows us to compute $Re_{\Lambda 0}$ from Re_{do} or vice versa, depending on available experimental data; or we can utilize the Re_{do} observed for incompressible wakes in order to predict the behavior of compressible flows, which we shall do below before we attempt numerical estimates.

IV. Extension to Compressible Wake

The compressible analog of Eq. (8) can be written by using the finding[10] that the velocity fluctuations on the wake axis remain the same as in Eq. (5), with the addition of a temperature factor:

$$u'(0) = C_1[u_e - u(0)](t+1)^{1/2} \tag{10}$$

where the "temperature defect" $t = [T(0) - T_e]/T_e$; subscripts of properties outside the wake have been changed here from "∞" to "e" to allow for inviscid gradients. Some sort of temperature-viscosity law is also needed and we shall use

$$\mu \sim T^k \tag{11}$$

The turbulence scale Λ maintains its value of Eq. (6) in the compressible case.[10] Combining the latter equation with Eqs. (3), (5), and (11) we obtain

$$Re_w = (Re_{\Lambda 0}/C_3)(t+1)^{-(k+1/2)} \tag{12}$$

or, since Re_o is the "critical" Reynolds number for the incompressible case,

$$Re_w = (u_e C_D h/\nu_e)_0 = Re_o(1+t)^{-(k+1/2)} \tag{13}$$

which is the compressible counterpart of Eq. (8). The temperature defect t must be evaluated here at a point downstream of transition, but still close enough to it so that turbulent diffusion has not yet taken its toll; thus, the quantity t can be approximated to the laminar value at the transition location if transition had not occurred. Since t is a function of the distance x from the body, we already detect in Eq. (13) a relation connecting the transition distance x_T from the body with the wake Reynolds number Re_w. Conversely, we see that in this case t and thus also x_T can take on a range of values, depending on what Re_w happens to be, confirming observation (1) of the Introduction. Furthermore, we know very well that t depends on the freestream (or flight) Mach number M_∞, the heat exchanged between flow and body Q, the ratio of specific heats γ, and the gas Prandtl number σ; that the edge unit Reynolds number $u_e(x)/\nu_e(x)$ and the momentum defect $C_D h$ can be computed from the inviscid and laminar viscous flow, and that C_1, C_2, and C_3 can be obtained from turbulence measurements. Thus, we can obtain from Eq. (13)

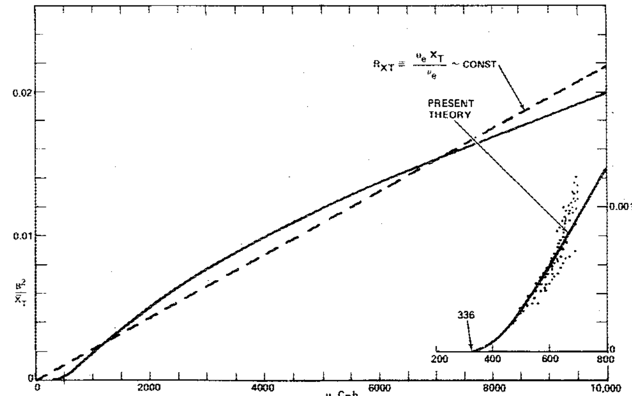

Fig. 3 The wake transition formula, Eq. (17), compared with the constant-Reynolds-number approximation. Data points in the inset are from Ref. 14.

an algebraic relation

$$x_T = x_T(M_\infty, Re, Q, \gamma \ldots; \quad Re_{\Lambda 0}) \tag{14}$$

which determines the transition distance behind the body. An example of this procedure, which is both illustrative and practically useful, will be given below.

V. Two-Dimensional Wake Not Too Close to the Body

In many instances, the portion of the compressible laminar wake between the shedding body and the transition zone satisfies the requirements for, and obeys the equations of, laminar similarity theory such as presented by Kubota.[11] In its two-dimensional formulation, this theory expresses t as

$$t = \frac{\sqrt{\sigma}(\gamma-1)(1+Q)M_e^2 Re_w}{4\sqrt{\pi}}\left(\frac{C_D h}{x}\right)^{1/2} \text{for } \frac{u_e - u(0)}{u_e} \ll 1 \tag{15}$$

where M_e is the local wake edge Mach number and

$$Q \equiv \frac{\bar{Q}}{\frac{1}{2}\rho_e u_e^3 C_D h} \tag{16}$$

is the nondimensional thermal power exchanged between the body and the flow; $Q<0$ when the body absorbs heat from and $Q>0$ when it rejects heat to the flow. Inserting Eq. (15) into (13) we get for the nondimensional transition distance

$$\bar{x}_T \equiv \frac{x_T}{C_D h} = \frac{\sigma(\gamma-1)^2 M_e^4 (1+Q)^2 Re_w}{16\pi}\left[\left(\frac{Re_w}{Re_o}\right)^{1/k+0.5} - 1\right]^{-2} \tag{17}$$

This equation not only supports qualitatively the experimental findings listed in the Introduction, but for the first time presents quantitatively the dependence of x_T on M_e, Q, Re_w, etc. It is seen that the transition zone moves downstream very fast as the Mach number increases. The x_T also increases with Prandtl number, ratio of specific heats, and body heating. Since k is of order 0.75, a Reynolds number increase moves the transition zone toward the body, as anticipated.

The physical constant present in Eq. (17) is the threshold turbulence Reynolds number $Re_{\Lambda 0}$, chosen purely for convenience to be represented by the critical Reynolds number Re_o for transition of an incompressible wake, via Eq. (8). Consider what this tells us about the minimum Reynolds number of transition for a cylinder of diameter d at high speeds. This is by definition the case $x_T = \infty$ obtained when the bracket in Eq. (17) is zero:

$$Re_w = u_e C_D h/\nu_e = Re_o \tag{18}$$

or

$$\frac{u_\infty}{\nu_\infty}\left(\frac{\nu_\infty}{u_\infty}\frac{u_e}{\nu_e}\frac{C_D h}{d}\right)d \equiv Re_{\infty d}\left(\frac{\nu_\infty}{u_\infty}\frac{u_e}{\nu_e}\frac{C_D h}{d}\right) = Re_o \tag{19}$$

At supersonic or hypersonic speeds, it is well known[1] that the quantity in parentheses is very small (order 0.01-0.1), whereas it is equally well known[12] that the minimum transition Reynolds number Re_o for a cylinder at low speeds is of order 300. Thus, Eq. (19) says that the minimum body Reynolds number $Re_{\infty d}$ at high speeds would be an order or two larger in magnitude than its low speed counterpart Re_o (depending on conditions), which confirms the results of Behrens[5] and Kendall[6] embodied in observation (3) of the Introduction.

VI. Evaluation of $Re_{\Lambda 0}$ and Re_o

Detailed numerical predictions of transition location via Eq. (17) cannot be made until Re_o or $Re_{\Lambda 0}$ is determined. So far, the existing literature contains no precise measurements of $Re_{\Lambda 0}$ as defined here but there are several works agreeing on $Re_o \approx 300$ as the threshold of appearance of turbulence in a low speed two-dimensional wake (see Ref. 12). A survey of turbulent wake studies[9,10,13] also gives the following added information: $C_1 \doteq 0.37$, $C_2 = 0.125$, $C_3 = 0.04625$. On the basis of our earlier definitions [see Eq. (5)], this says that the minimum turbulence Reynolds number for the self preservation of turbulence is $300 \times 0.04625 \simeq 14$. This author also elected to disqualify his Mach 3 data of Ref. 14 for comparison with the present transition criterion, so that he could use them to provide additional data toward the value of $Re_{\Lambda 0}$. Solving Eqs. (5) and (17) for $Re_{\Lambda 0}$, the data of Ref. 14 were used to provide an average $Re_{\Lambda 0}$ of 15.5; the comparison with the previous estimate of 14 is shown on Fig. 2. The value of $Re_{\Lambda 0} = 15.5$ was used for subsequent calculations.

Students of turbulence have long been admonished that the minimum permissible value of $Re_{\Lambda 0}$ is "much larger than unity," whereas this author has observed self-preserving wake turbulence for $Re_\Lambda \simeq 40$. That the $Re_{\Lambda 0}$ found herein lies between these two values is therefore not surprising. Quite independently, Finson[15] has plotted the effect of Re_Λ on dissipation as measured by a number of workers and observes that turbulence below the range $10 < Re_\Lambda < 30$ is marked by rapid dissipation in decaying grid flow.

VII. Numerical Evaluation of the Transition Criterion

Using an exponent of $k = 0.75$ for the temperature-viscosity relation, the transition formula (17) has been plotted on Fig. 3. The ordinate has been written as $B^2/\bar{x}_T$, where

$$B \equiv \frac{\sqrt{\sigma}(\gamma - 1) M_e^2 (1 + Q)}{4\sqrt{\pi}} \tag{20}$$

Because $Re_{\Lambda 0} = 15.5$ was adopted as noted above, the critical Re_o for the appearance of turbulence is now 335 instead of 300. At that point $\bar{x}_T = \infty$ (or $B^2/\bar{x}_T = 0$). All the transition data reported by this author in Refs. 10, 14, and 16 are also plotted on Fig. 3; although the apparent coincidence between points and curve is prejudiced by the use of the same data for finding $Re_{\Lambda 0}$ (see Fig. 2), the scatter of the data is considerably smaller than in earlier groupings (see Fig. 2 of Ref. 14). Furthermore, the agreement would be almost just as good if $Re_{\Lambda 0}$ was taken directly from Roshko's 1954 data.[12]

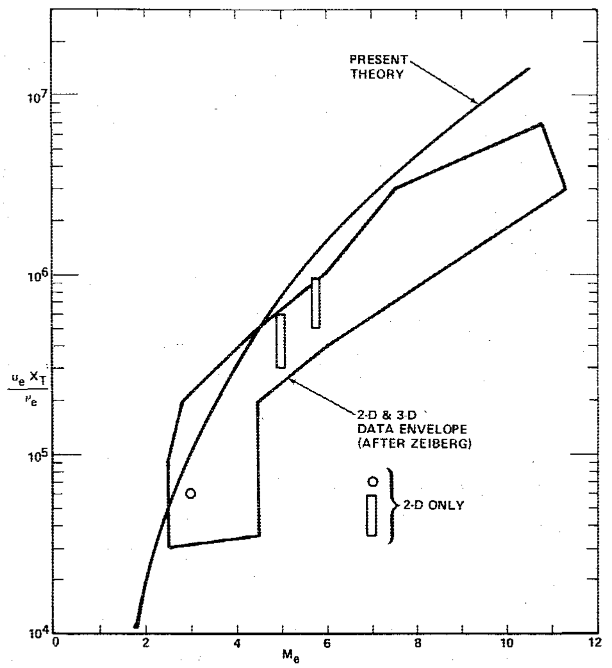

Fig. 4 Mach number dependence of the transition distance as predicted by an approximate form of Eq. (17) (the dashed line on Fig. 3). The two-dimensional data shown are from cylinders at $M_\infty \simeq 6$.

There is an ulterior motive behind the choice of the ordinate in Fig. 3. One can draw a straight line through the origin that is a good approximation of the present theory over a wide range. The equation of such a straight line would be

$$Re_{x_T} \equiv u_e x_T / \nu_e = fn(B) = fn(M_e, Q, \gamma, \sigma) \tag{21}$$

Observation (4) of the Introduction, drawn from adiabatic air wakes at constant Mach number, is then borne out that Re_{xT} is "constant." The contention here is that these early data belonged to the solid rather than the dashed line of Fig. 3 all along, a point easily obscured by the data scatter in the earlier experiments.

VIII. Further Comparison with Experiment

As it should, Eq. (17) utilizes flow features downstream of the body that differ from the stream properties because of inviscid gradients caused by the presence of the body itself; to use this formula, therefore, we require knowledge of the inviscid and laminar viscous wake flow. Unfortunately, the transition data reported earlier do not often supply this information, so that their comparison with Eq. (17) would require the usual inviscid flowfield computations. Expedients are possible, however, for such comparisons as involving, for example, the Mach number effect. For this purpose, we can use the straight line approximation of Fig. 3

$$\bar{x}_T / B^2 \simeq 5 \times 10^5 Re_w^{\ -1} \tag{22}$$

in which B can be further evaluated for adiabatic air wakes ($\gamma = 1.4$, $Q = 0$, $\sigma = 0.75$) as

$$B^2 = 0.00238 M_e^2 \tag{23}$$

Inserted in Eq. (22) this gives

$$Re_{xT} \equiv u_e x_T / \nu_e \simeq 1200 M_e^4 \tag{24}$$

The coordinates in this equation are identical to those used by Zeiberg[17] in plotting the wake transition data shown on Fig. 4, which also plots Eq. (24). The agreement is good.

A final test of Eq. (17) can be made by checking the effect of heat transfer. According to this equation

$$\frac{x_T}{x_T(Q=0)} = (1+Q)^2 \tag{25}$$

which, in Fig. 5, shows similarly encouraging resemblance to experimental data, in this case taken from Ref. 14.

Concluding Remarks

In the foregoing, we have approached the prediction of wake transition from a new direction: we predict transition onset by excluding the circumstances where transition is forbidden by viscous dissipation. The one really novel element in this approach is to postulate that transition will occur *as soon as* dissipation allows it. We then use knowledge of both the similar laminar wakes and self-preserving turbulent wakes to express the criterion quantitatively. A minimum-turbulence Reynolds number of about 15 is established, which lies in the expected range. This "dissipation" approach to predicting transition onset displays correctly all the trends found in experiments, such as the effect on transition distance of Mach number, Reynolds number, and heat transfer. It explains convincingly the earlier observation of a "constant" transition Reynolds number and of a minimum transition Reynolds number at supersonic and hypersonic speeds.

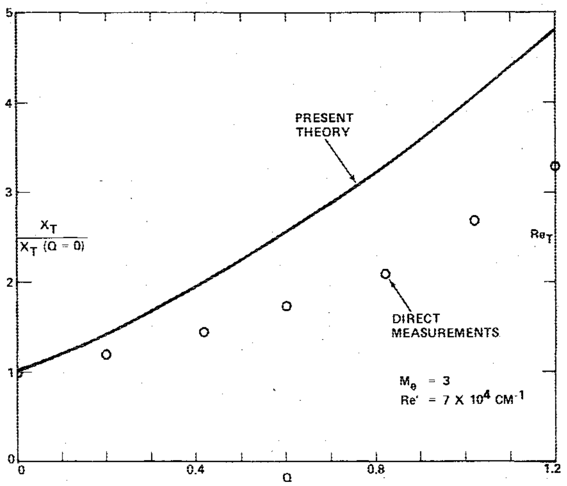

Fig. 5 Test of the heat transfer effect on wake transition. Data are from Ref. 14.

Quantitatively, it agrees with all existing data, especially if one considers experimental scatter, the perennial controversy of what constitutes transition and the naturally diffuse boundaries of transition.

The present approach is so simply and categorically expressed by Eq. (4) that it seems to leave no room for the eventuality of flows where transition is delayed or accelerated by artificial means; there are reports,[18] for example, of "tripping" a wake by acoustic excitation. One might also argue that in a wind tunnel with unusually or ideally "quiet" flow (no stream disturbances whatsoever) transition to turbulence may be greatly delayed. A brief response to this is that there are no actual measurements, so far, that clearly show a violation of Eq. (4) in the case of wake transition "prematurely" accelerated by acoustic means. For example, it might well be that u' or Λ in these cases is also larger than normal, so that Eq. (4) remains valid; the necessary measurements have not been done. (Sato and Kuriki,[18] in fact, report that the premature wake "turbulence" they generated by acoustic radiation differed considerably from that generated naturally.)

Nor does the present approach compete with bona fide theories of turbulence generation mechanisms such as hydrodynamic stability theory; the comparison itself is not relevant since the present approach merely establishes thresholds for allowable turbulence intensity and scale. The pivotal point in the present scheme is Eq. (4); even if a quantitative link is someday established between stability theory and transition, the former is forbidden from violating any such threshold as represented by Eq. (4). The crucial role of stability theory, on the other hand, is to see if there are flows for which transition can occur much later than is allowed by viscous turbulence dissipation. In the meantime, note that the present approach has already much in common with hydrodynamic stability theory since they both deal with the damping effect of viscosity on secondary motion.

A point worth mentioning is the indirect evidence supplied by the apparent success of the present method toward the validity of earlier wake turbulence measurements and their generalization, exemplified by Eq. (10). The compressibility scaling represented in this equation by the temperature factor $(t+1)^{1/2}$, together with the exponent k of the temperature-viscosity relation, supplies the distinctive feature of compressible wake transition: that the hotter the wake (whether driven by flight Mach number or heat addition), the longer transition has to "wait" for viscosity to allow turbulent motion. The one-half power exponent of the temperature factor is quite crucial, and its absence or change would have profound effects on the Reynolds number dependence of the transition location. The author owes much to the original proposals of Morkovin[19] for their physical insight into compressible turbulence scaling, which originally[13] led to the statement of Eq. (10) and its support by measurement.

Finally, the reader can detect a number of possible improvements for future use of the threshold criterion in free shear flows; for example, the algebraic linking of post-transition properties with the *laminar* velocity defect is, at best, a serviceable approximation. Also, the wake axis properties have been used throughout, whereas it is known that transition first appears at off-axis maximum shear zones. Such improvements are not vital to the understanding of the physical picture, but may well result in closer interpretation of future experiments on wake transition.

Acknowledgment

This work was performed under Contract AFOSR F44620-75-C-0016.

References

[1] Lees, L., "Hypersonic Wakes and Trails," *AIAA Journal*, Vol. 2, March 1964, pp. 417-428.

[2] Burnage, H., "Contribution A L'Etude Du Sillage En Transition D'Un Cylindre Perpendiculaire A Un Ecoulement Supersonique," Universite de Provence, France, 1973.

[3] Waldbusser, E., "Shape Effects on Hypersonic Slender Body Wake Geometry and Transition Distance," *Journal of Spacecraft and Rockets*, Vol. 4, May 1967, pp. 657-662.

[4] Finson, M. L. and Legner, H. H., "Mach Number Dependence of Laminar Near-Wake Transition for Slender Cones," *AIAA Journal*, Vol. 15, March 1977, pp. 417-419.

[5] Behrens, W., "Far Wake Behind Cylinders at Hypersonic Speeds: I. Flowfield," *AIAA Journal*, Vol. 5, Dec. 1967, pp. 2135-2141.

[6] Kendall, J. M. Jr., "Experimental Study of Cylinder and Sphere Wakes at a Mach Number of 3.7," Jet Propulsion Laboratory, California Institute of Technology, TR 32-367, Nov. 1962.

[7] Demetriades, A. and Gold, H., "Transition to Turbulence in the Hypersonic Wake of Blunt-Bluff Bodies," *ARS Journal*, Vol. 32, Sept. 1962, pp. 1420-1421.

[8] Gold, H., "Stability of Laminar Wakes," Ph.D. Thesis, California Institute of Technology, Pasadena, Calif., 1963.

[9] Townsend, A. A., *The Structure of Turbulent Shear Flow*, Cambridge University Press, Cambridge, 1956, Chapt. 7.

[10] Demetriades, A., "Turbulence Measurements in a Supersonic Two-Dimensional Wake," *Physics of Fluids*, Vol. 13, July 1970, pp. 1672-1678.

[11] McCarthy, J. F., Jr. and Kubota, T., "A Study of Wakes Behind a Circular Cylinder at $M=5.7$," *AIAA Journal*, Vol. 2, April 1964, pp. 629-636.

[12] Roshko, A., "On the Development of Wakes from Vortex Streets," NACA TR 1191, Washington, D.C., 1954.

[13] Demetriades, A., "Turbulence Measurements in an Axisymmetric Compressible Wake," *Physics of Fluids*, Vol. 11, Sept. 1968, pp. 1841-1852.

[14] Demetriades, A., "Heat Transfer Effects on Supersonic Wake Transition," *Physics of Fluids*, Vol. 13, No. 1, Jan. 1970, pp. 204-206.

[15] Finson, M. L., "Hypersonic Wake Aerodynamics at High Reynolds Numbers," AIAA Paper 72-701, New York, N.Y., 1972.

[16] Demetriades, A., "Observations on the Transition Process of Two-Dimensional Supersonic Wakes," *AIAA Journal*, Vol. 9, Nov. 1971, pp. 2128-2134.

[17] Zeiberg, S., "Transition Correlations for Hypersonic Wakes," *AIAA Journal*, Vol. 2, No. 3, March 1964, pp. 564-565.

[18] Sato, H. and Kuriki, K., "The Mechanisms of Transition in the Wake of a Thin Flat Plate Placed Parallel to a Uniform Flow," *Journal of Fluid Mechanics*, Vol. II, Part 3, Nov. 1961, pp. 321-352.

[19] Morkovin, M. V., "Effects of Compressibility on Turbulent Flows," Coll. International Du Centre National de la Recherche Scientifique, No. 108: Mechanique de la Turbulence, Editions Du Centre National de la Recherche Scientifique, Paris, 1962, p. 367.